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11. A NOTE OF TWO-DEMENSIONAL LANDMARK-BASED OBJECT RECOGNITION/ON LANDMARK-BASED SHAPE ANALYSIS/THE PASM PARALLEL PROCESSING SYSTEM: HARDWARE DESIGN AND INTELLIGENT OPERATING SYSTEM CONCEPTS/EXPERT SYSTEMS FOR THE SCHEDULING OF IMAGE PROCESSING TASKS ON A PARALLEL PROCESSING SYSTEM

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AN APPLICATION OF TENSOR THEORY TO 3-D SHAPE ANALYSIS†

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Abstract

In this paper, we present two new approaches to address the shape recognition problem. The first concept is an extension of 2-D landmark-based shape analysis to the 3-D case. Secondly, we extend the work of Cyganski and Orr to rigid non-planar objects. Both approaches will be mathematically formulated, however no experimental results will be shown in this paper.

I. 3-D Landmark-based Shape Analysis

Objects can be represented by landmarks or tokens in 3-D space. Landmarks or tokens are points, usually defined along the object surface, that have important shape attributes. These attributes are chosen a priori and are problem dependent. Example of landmarks are corners and points of high curvature. We shall assume throughout this paper that the landmark locations have been detected. We shall address the problem of processing the landmarks to identify the object. To identify an observed (unknown) object from a library of objects, we find the necessary transformation that maps landmarks of each library object to the corresponding landmarks of the observed object. It is difficult, even if it exists, to determine this transformation. If the transformation can be found, then shape features based on the transformation can be extracted and used for object identification. We shall address this problem similar to the way we addressed the problem of 2-D shape analysis [AnD85]. For 2-D shape analysis the object was partitioned into triangles and the mapping used was triangular deformation obtained through an affine transformation. The shape features in the 2-D case were based on the dilatations obtained from the transformation. In 3-D we partition the object into tetrahedra and use the tetrahedral deformation, i.e. a transformation from one tetrahedron to another.

I.1. Quasiconformal mapping

Tetrahedral deformation can be determined by a 3-D affine transformation. The 3-D affine transformation is a class of quasiconformal mappings in 3-D which are defined [Vai61] as follows:
A diffeomorphism, $f:\Omega \rightarrow \bar{\Omega}$, $(\Omega, \bar{\Omega} \subset R^3)$ is called K-quasiconformal if

$$\sup_{x \in \Omega} D(x) \le K < \infty \quad \text{for some constant K,}$$

$$\text{where} \quad D(x) = \max(\bar{D}(x), \underline{D}(x)) ,$$

$$\bar{D}(x) = \frac{\left(\max_{|e|=1} |f'(x)e|\right)^3}{|J(x)|} , \qquad \underline{D}(x) = \frac{|J(x)|}{\left(\min_{|e|=1} |f'(x)e|\right)^3} ,$$

$J(x)$ and f' are the Jacobian and the derivative of f respectively. Note that $\max_{|e|=1} |f'(x)e|$ and $\min_{|e|=1} |f'(x)e|$ correspond to the largest and the smallest eigenvalue of the Jacobian matrix of f. $\bar{D}(x)$, $\underline{D}(x)$, and $D(x)$ are known as the outer, inner and maximal dilatations of f, and are related to the eigenvalues of the Jacobian matrix of f by the above equations. Infinitesimally f maps a ball to an ellipsoid. These dilatations at each $x \in \Omega$ are various combinations of the ratios of the semi-axes of the 3-D ellipsoid. In general, it is difficult (if it exists) to find a diffeomorphic map of a library object to an observed object. The affine transformation from one tetrahedron to another is however uniquely determined. Since the 3-D affine transformation is a class of quasiconformal mappings, we can use the above definitions to obtain the dilatations. The maximal dilatation derived from the affine transformation can be used as a distortion measure between the two tetrahedra (they are similar if the maximal dilatation is 1). Using an approach similar to that of partitioning a 2-D object into triangles, we partition a 3-D object into tetrahedra. We then evaluate the maximal dilatation and the scale change (magnitude of the Jacobian) of each tetrahedral deformation between a library object and the observed object. If the two objects are similar, for each tetrahedral deformation between the two objects, the maximal dilatation is one and the scale change remains constant. Hence, the standard deviations of the maximal dilatation and the scale change associated with all the tetrahedral deformations between the two objects can be used to determine how similar the two objects are. The smaller the deviations are, the more similar the two objects are.

I.2. Strain tensors

The above concept can similarly be formulated by tensor analysis. We assume that the observed object is deformed from a library object. Denote Ω (with respect to reference frame X, x^i, $i=1,2,3$) as the region occupied by the library object, and $\bar{\Omega}$ (with respect to reference frame $\bar{X}$,

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$\bar{x}^i$, i=1,2,3) as that of the observed object. We assume that the deformation from Ω to $\bar\Omega$ is homeomorphic. Assuming both X and $\bar{X}$ are orthogonal cartesian, the square of the arc length of the library and observed object using the general summation rule, are respectively

$ds^2 = dx^i dx^i$ and $d\bar{s}^2 = d\bar{x}^i d\bar{x}^i$ $\quad i=1,2,3.$

Thus, $d\bar{s}^2 - ds^2 = 2\mu_{ij} dx^i dx^j \quad i,j=1,2,3.$ and $\mu_{ij} = \frac{1}{2}\left(\frac{\partial \bar{x}^k}{\partial x^i}\frac{\partial \bar{x}^k}{\partial x^j} - \delta_{ij}\right).$

The tensor μ_{ij}, which is symmetric, is called the strain tensor [Sok51]. The eigenvalues of μ_{ij} are called the principal strains, and the corresponding eigenvectors are orthogonal. The directions corresponding to the principal strains are called the principal directions (axes) of the strain tensor. At any given point, it is a mapping from a ball to an ellipsoid. The ratio of each semi-axis of the ellipsoid to the diameter of the ball equals $\sqrt{1+2\lambda_i}$, i=1,2,3, where λ_i's are the eigenvalues of μ_{ij}. Again, the deformation from a library object to the observed object may not be homeomorphic. It is difficult to determine the deformation function even if it is homeomorphic. The tetrahedral deformation used for the above landmark-based problem can however be uniquely determined. The various dilatations (inner, outer, maximal) of the tetrahedral deformation can also be easily computed by means of the eigenvalues of the strain tensor associated with the tetrahedral deformation. The strain tensor analysis which provides an intuitive analysis of deformation thus can also be used to solve the landmark-based problem.

II. Surface tensors

We shall develop another concept for object identification dealing with objects that are characterised by surface patches rather than sets of landmarks. Object identification becomes a problem of identifying surface patches. Let Ω with coordinate reference frame X be a surface occupied by a library object, and $\bar\Omega$ with coordinate reference frame $\bar{X}$ be a surface of an observed object. This notation is consistent with the strain tensor notation presented above, the difference being that Ω is a surface instead of a set of landmarks. We further assume that Ω and $\bar\Omega$ are related by an affine transformation, i.e. $\bar{x}^i = a^i_j(x^j - c^j) + \bar{c}^i$, where c^i and $\bar{c}^i$ are the centers of gravity of Ω and $\bar\Omega$. Here, we are interested in finding the a^i_j, the coefficients of the affine transformation. We can thus transform the library object (surface) with respect to the observed object (surface) by the affine transformation. The similarity between the library object (surface) and the observed object (surface) is determined by how the transformed library object (surface) is correlated with the observed object (surface). We describe how to determine the affine coefficients below. We form a surface (moment) tensor on Ω and $\bar\Omega$ as follows:

$$S^{ij\cdots} = \int_\Omega (x^i-c^i)(x^j-c^j)\cdots dx^1dx^2dx^3 = \int (x^i-c^i)(x^j-c^j)\cdots I_\Omega(x^1,x^2,x^3)dx^1dx^2dx^3$$

$$\bar{S}^{ij\cdots} = \int_{\bar\Omega} (\bar{x}^i-\bar{c}^i)(\bar{x}^j-\bar{c}^j)\cdots d\bar{x}^1d\bar{x}^2d\bar{x}^3 = \int (\bar{x}^i-\bar{c}^i)(\bar{x}^j-\bar{c}^j)\cdots I_{\bar\Omega}(\bar{x}^1,\bar{x}^2,\bar{x}^3)d\bar{x}^1d\bar{x}^2d\bar{x}^3$$

$$= \int_\Omega a^i_k(x^k-c^k)a^j_l(x^l-c^l)\cdots |J|\, dx^1dx^2dx^3 = a^i_k a^j_l \cdots |J| S^{ij\cdots}$$

$$\text{where } I_\Omega(x^1,x^2,x^3) = \begin{cases} 1 & \text{if } (x^1,x^2,x^3) \in \Omega \\ 0 & \text{otherwise} \end{cases}, \quad I_{\bar\Omega}(x^1,x^2,x^3) = \begin{cases} 1 & \text{if } (\bar{x}^1,\bar{x}^2,\bar{x}^3) \in \bar\Omega \\ 0 & \text{otherwise} \end{cases}$$

$$a^i_j = \frac{\partial \bar{x}^i}{\partial x^j} \qquad |J| = \left|\det\left(\frac{\partial \bar{x}^i}{\partial x^j}\right)\right|$$

Thus, $S^{ij\cdots}$ is a relative tensor of weight -1. Following the above steps we form various orders of surface tensors for both Ω and $\bar\Omega$, we then contract them to unit rank absolute tensors similar to [CyO85]. Three pairs of such unit rank absolute tensors of Ω and $\bar\Omega$, say, u^i, $\bar{u}^i$, v^i, $\bar{v}^i$, w^i, $\bar{w}^i$, are required to form a system of linear equations:

$\bar{u}^i = a^i_j u^j, \quad \bar{v}^i = a^i_j v^j, \quad \bar{w}^i = a^i_j w^j$

such that we can obtain a^i_j.

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